Large logarithms at subleading power

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Expansion of x-section

The cross section can be expanded in a series of a small variable τ,

$$\sigma(\tau) = C\delta(\tau) + \sum_n \alpha_s^n \left[\frac{\ln^{2n-1}\tau}{\tau} + \underbrace{\ln^{2n-1}\tau}_{NLP} + \cdots \right] \qquad (1)$$

Here τ can be the N-jettiness variable, the threshold variable $1 - M^2/s$, the transverse momentum of a lepton pair q_T, the mass ratio m_h^2/m_b^2, $\cdots$

1. Phenomenology: useful for NN(N)LO differential calculations in q_T/N-jettiness slicing methods [Moult, Rothen, Stewart, Tackmann, Zhu 16', Boughezal, Liu, Petriello,16']
2. Theory: NLP factorization and resummation [Bonocore, Laenen, Magnea, Melville, Vernazza, White, 15', 16', Moult, Stewart, Vita, Zhu, 18', Beneke, Broggio, Garny, Jaskiewicz, Szafron, Vernazza, and JW, 18', Laenen, Damste, Vernazza, Waalewijn, Zoppi, 20', see below for more refs.]
3. Amplitude: general structure, soft theorem [Rodina 19']

Improvement for subtraction

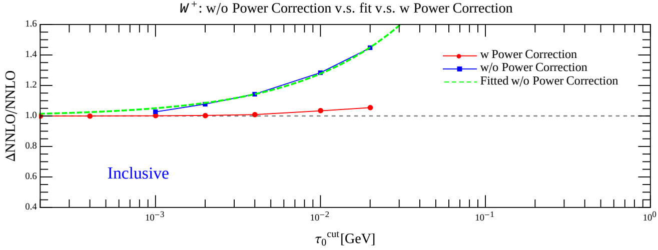

Figure: $O(\alpha_s^2)$ correction for DY production with N-jettiness subtraction from 1612.02911

Without the power corrections, τ_{cut} should be set to below 10^{-3}GeV to reproduce the exact NNLO coefficient. The cut can be relaxed by a factor of 10 when the power corrections are included.

Recent development

- Beyond leading logarithms (at $O(\alpha_s)$) [Boughezal, Isgro, Petriello, 18', Ebert, Moult, Stewart, Tackmann, Vita, Zhu, 18']
- Beyond $2 \to 1$ or $1 \to 2$ [Beekveld, Beenakker, Laenen, White 19',Boughezal, Isgro, Petriello, 19']
- Threshold/Thrust resummation at NLP [Moult, Stewart, Vita, Zhu, 18', Beneke, Broggio, Garny, Jaskiewicz, Szafron, Vernazza, and JW, 18', Bahjat-Abbas, Bonocore, Damste, Laenen, Magnea, Vernazza, White 19', Ajjath, Mukherjee, Ravindran 20']
- Rapidity divergences in q_T spectrum or energy-energy correlators [Ebert, Moult, Stewart, Tackmann, Vita, Zhu, 18', Moult, Vita, Yan, 19']
- Soft quark Sudakov [Liu, Penin, 17', Moult, Stewart, Vita, Zhu, 19', Liu, Mecaj, Neubert, Wang, Fleming, 20', JW, 20']
- Subleading power effects in B physics and heavy quarkonium production [Ma, Qiu, Sterman, Zhang 13', Lee, Sterman 20', Li, Lü, Sheng Wang, Wang, Wei, 17',20']

An example

Energy-Energy Correlator is defined by

$$\mathrm{EEC}(\chi) = \sum_{a,b} \int d\sigma_{V\to a+b+X} \frac{2E_a E_b}{Q^2 \sigma_{\mathrm{tot}}} \delta(\cos\theta_{ab} - \cos\chi) \tag{2}$$

The NLP result is [Moult, Vita, Yan, 19']

$$\begin{aligned}\mathrm{EEC}^{(2)} &= -\sqrt{2a_s} D[\sqrt{2a_s} \ln(1-z)], \quad a = \alpha_s/(4\pi) C_A \\ &= \sum_{n=0} \frac{(-1)^{n+1} 2^{2n+1}}{(2n+1)!!} a_s^{n+1} \ln(1-z)^{2n+1}\end{aligned} \tag{3}$$

with

$$z = \frac{1-\cos\chi}{2}, \quad D(x) = \frac{1}{2}\sqrt{\pi} e^{-x^2} \mathrm{erfi}(x) = e^{-x^2} \int_0^x dt\ e^{t^2} \tag{4}$$

The origin of large logarithms

In the soft limit $k^\mu \to 0$, (LBK/soft theorem)

$$A(k,\{p_i\}) = \sum_i (-g_s)\mathbf{T}_i \left(\frac{\varepsilon(k)\cdot p_i}{k\cdot p_i} + \frac{\varepsilon_\mu k_\nu J_i^{\mu\nu}}{k\cdot p_i} \right) A_0(\{p_i\}) \quad (5)$$

with

$$J_i^{\mu\nu} = p_i^\mu \frac{\partial}{\partial p_{i\nu}} - p_i^\nu \frac{\partial}{\partial p_{i\mu}} + \Sigma_i^{\mu\nu} \quad (6)$$

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Integrating over the constrained phase space,

$$\int d^d k \delta(k^2)\theta(k^0) \frac{1}{k\cdot p_i}\frac{1}{k\cdot p_j} f(k) \quad (7)$$

$$\frac{1}{\epsilon}\tau^\epsilon = \frac{1}{\epsilon} + \ln\tau \quad (8)$$

$$\frac{1}{\epsilon^2}\tau^\epsilon = \frac{1}{\epsilon^2} + \frac{\ln\tau}{\epsilon} + \frac{1}{2}\ln^2\tau \quad (9)$$

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Controlled by anomalous dimensions of certain effective operators

At leading power, up to the two-loop level,

$$\mathbf{\Gamma} = -\gamma_{\rm cusp}(\alpha_s) \sum_{i<j} \mathbf{T}_i \cdot \mathbf{T}_j \ln\left(\frac{-s_{ij}}{\mu^2}\right) + \sum_i \gamma_i(\alpha_s) \tag{10}$$

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$$\frac{d}{d\ln\mu} C_V(-M^2, \mu) = \left[C_F \gamma_{\text{cusp}} \ln\frac{-M^2}{\mu^2} + \gamma_V \right] C_V(-M^2, \mu) \tag{11}$$

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Next-to-leading power?

Subleading power operators

$$\mathcal{L}_{\mathrm{SCET}} = \sum_{i=1}^{N} \mathcal{L}_i(\psi_i, \psi_s) + \mathcal{L}_s(\psi_s) \tag{12}$$

The general structure of subleading operators

$$J = \int dt \; C(\{t_{i_k}\}) \, J_s(0) \prod_{i=1}^{N} J_i(t_{i_1}, t_{i_2}, \dots) \tag{13}$$

where

$$J_i(t_{i_1}, t_{i_2}, \dots) = \prod_{k=1}^{n_i} \psi_{i_k}(t_{i_k} n_{i+}) \tag{14}$$

with gauge-invariant collinear "building blocks"

$$\psi_i(t_i n_{i+}) \in \begin{cases} \chi_i(t_i n_{i+}) \equiv W_i^\dagger \xi_i & \text{collinear quark} \\ \mathcal{A}^\mu_{\perp i}(t_i n_{i+}) \equiv W_i^\dagger [iD^\mu_{\perp i} W_i] & \text{collinear gluon} \end{cases} \tag{15}$$

Subleading power operators

LP:

$$J_i^{A0}(t_i) = \psi_i(t_i n_{i+}) \,. \tag{16}$$

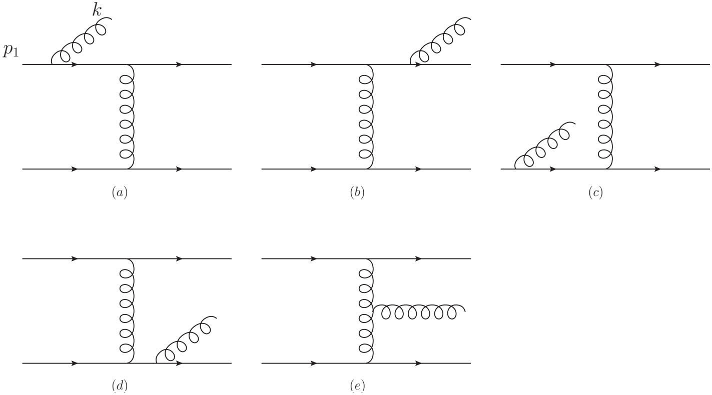
NLP $[O(\lambda), O(\lambda^2)]$:

- $i\partial_\perp \quad \to J^{A1} = i\partial_\perp J^{A0}$
- $in_- D_s \equiv in_-\partial + g_s n_- A_s \quad \to$ eliminated by E.o.M
- more building blocks $\to J^{B1} = \psi_{i_1}(t_{i_1} n_{i+})\psi_{i_2}(t_{i_2} n_{i+})$
- new building blocks, e.g., $n_-\mathcal{A} \quad \to$ eliminated by E.o.M
- pure soft sector J_s, e.g., $q \sim O(\lambda^3), F_s^{\mu\nu} \sim O(\lambda^4)$, not needed at NLP
- time-ordered product operators

$$J_i^{T1}(t_i) = i \int d^4x \, \mathbf{T}\left\{ J_i^{A0}(t_i), \mathcal{L}_i^{(1)}(x) \right\} \tag{17}$$

LBK Theorem

Consider a process $ud \to ud + g$.



$$A(k, \{p_i\}) = \sum_i (-g_s)\mathbf{T}_i \left(\frac{\varepsilon(k) \cdot p_i}{k \cdot p_i} + \frac{\varepsilon_\mu k_\nu J_i^{\mu\nu}}{k \cdot p_i} \right) A_0(\{p_i\}) \quad (18)$$

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$$J_i^{\mu\nu} = p_i^\mu \frac{\partial}{\partial p_{i\nu}} - p_i^\nu \frac{\partial}{\partial p_{i\mu}} + \Sigma_i^{\mu\nu} \quad (20)$$

We expand the propagators in diagram (a)

$$\frac{(\not{p}_1 - \not{k})\not{\epsilon}}{(p_1-k)^2} = \frac{p_1\cdot\epsilon}{-p_1\cdot k} + \frac{i\Sigma^{\mu\nu}\epsilon_\mu k_\nu}{-p_1\cdot k} \quad (21)$$

$$\frac{1}{(p_1-p_3-k)^2} = \frac{1}{(p_1-p_3)^2} - k\cdot\frac{\partial}{\partial p_1}\frac{1}{(p_1-p_3)^2} \quad (22)$$

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Where is the blue part?

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Where is the blue part? It comes from diagram (e),

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$$J_i^{\mu\nu} = {\color{red} p_i^\mu \frac{\partial}{\partial p_{i\nu}}} {\color{blue} - p_i^\nu \frac{\partial}{\partial p_{i\mu}}} + \Sigma_i^{\mu\nu} \quad (20)$$

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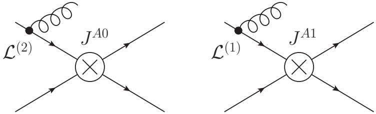
Where is the blue part? It comes from diagram (e), or from gauge invariance.

LBK Theorem



We reproduce LBK theorem with two time-ordered products

$$\int d^4x \mathbf{T}\{J^{A0}, \mathcal{L}^{(2)}(x)\}, \quad \int d^4x \mathbf{T}\{J^{A1}, \mathcal{L}^{(1)}(x)\}$$

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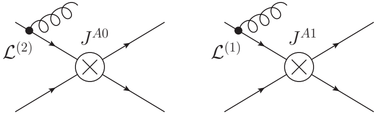

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No operators with soft fields needed!
No Ward identity needed!
J^{A1} is related to J^{A0}.

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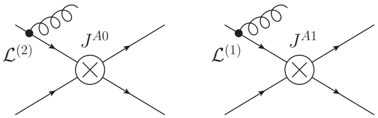

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J^{A1} is related to J^{A0}.
The LBK theorem has also been proven in the framework of label-SCET, [Larkoski, Neill, Stewart 14']. different operators, different propagators, different vertices
The two formulations of SCET recover the LBK formula in rather different ways.

Anomalous dimensions

With the definition $\mathbf{\Gamma} \equiv -\left(\frac{d}{d\ln\mu}\mathbf{Z}\right)\mathbf{Z}^{-1}$,

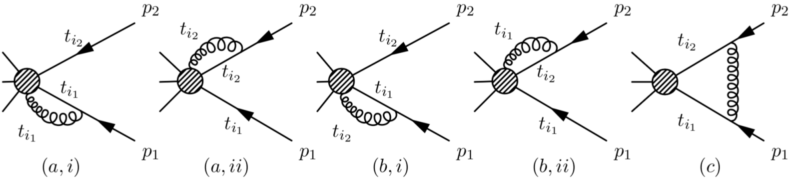
$$\Gamma_{PQ}(x,y) = \delta_{PQ}\delta(x-y)\left[-\gamma_{\rm cusp}(\alpha_s)\sum_{i<j}\sum_{k,l}\mathbf{T}_{i_k}\cdot\mathbf{T}_{j_l}\ln\left(\frac{-s_{ij}x_{i_k}x_{j_l}}{\mu^2}\right)+\sum_i\sum_k\gamma_{i_k}(\alpha_s)\right] + 2\sum_i\delta^{[i]}(x-y)\gamma^i_{PQ}(x,y)+2\sum_{i<j}\delta(x-y)\gamma^{ij}_{PQ}(y)\,. \quad (23)$$

In the calculation, we have used offshellness to regularize the IR poles, and found that they cancel between the soft and collinear contributions.

$$\mathbf{\Gamma} = \begin{pmatrix} \Gamma_{PQ} & \Gamma_{PT(Q')} \\ \Gamma_{T(P')Q} & \Gamma_{T(P')T(Q')} \end{pmatrix} = \begin{pmatrix} \Gamma_{PQ} & 0 \\ \Gamma_{T(P')Q} & \Gamma_{P'Q'} \end{pmatrix} \quad (24)$$

$$\gamma_{\rm cusp}(\alpha_s) = \frac{\alpha_s}{\pi} \quad \text{and} \quad \gamma_{i_k}(\alpha_s) = \begin{cases} -\dfrac{3\alpha_s C_F}{4\pi} & \text{(q)} \\ 0 & \text{(g)} \end{cases} \quad (25)$$

$$\gamma^i = \begin{pmatrix} \gamma^i_{PQ} & 0 \\ 0 & \gamma^i_{P'Q'} \end{pmatrix}, \qquad \gamma^{ij} = \begin{pmatrix} 0 & 0 \\ \gamma^{ij}_{T(P')Q} & 0 \end{pmatrix}. \quad (26)$$



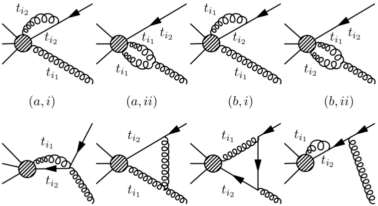
$$\gamma^{i}_{\chi_\alpha\chi_\beta,\chi_\gamma\chi_\delta}(x,y) = \frac{\alpha_s \mathbf{T}_{i_1}\cdot\mathbf{T}_{i_2}}{2\pi}\Bigg\{\delta_{\alpha\gamma}\delta_{\beta\delta}\Bigg(\theta(x-y)\left[\frac{1}{x-y}\right]_+ + \theta(y-x)\left[\frac{1}{y-x}\right]_+ - \theta(x-y)\frac{1-\frac{\bar{x}}{2}}{\bar{y}} - \theta(y-x)\frac{1-\frac{x}{2}}{y}\Bigg) - \frac{1}{4}\left(\sigma_\perp^{\nu\mu}\right)_{\alpha\gamma}\left(\sigma_{\perp\nu\mu}\right)_{\beta\delta}\left(\theta(x-y)\frac{\bar{x}}{\bar{y}} + \theta(y-x)\frac{x}{y}\right)\Bigg\}. \tag{27}$$

B1 to B1 with F=1

$$
\begin{aligned}
\gamma^{i}_{\mathcal{A}^\mu\chi_\alpha,\mathcal{A}^\nu\chi_\beta}(x,y) \quad = \quad & \frac{\alpha_s \mathbf{T}_{i_1}\cdot\mathbf{T}_{i_2}}{2\pi}\left\{ g_\perp^{\mu\nu}\delta_{\alpha\beta}\left(\theta(x-y)\left[\frac{1}{x-y}\right]_+ + \theta(y-x)\left[\frac{1}{y-x}\right]_+\right.\right. \\
& - \frac{\theta(x-y)}{\bar{y}}\left(1+\frac{\bar{x}(\bar{x}+\bar{y})}{2x}\right) - \frac{\theta(y-x)}{2y}(\bar{x}+\bar{y})\Bigg) \\
& \left. + \frac{1}{4}\left([\gamma_\perp^\mu,\gamma_\perp^\nu]\right)_{\alpha\beta}(x+y)\bar{x}\left(\frac{\theta(x-y)}{\bar{y}x}+\frac{\theta(y-x)}{y\bar{x}}\right)\right\} \\
& - \frac{\alpha_s(\mathbf{C_F}+\mathbf{T}_{i_1}\cdot\mathbf{T}_{i_2})}{4\pi}\left\{ g_\perp^{\mu\nu}\delta_{\alpha\beta}\left(\frac{\theta(x-\bar{y})\bar{x}}{yx}(\bar{x}+\bar{y})+\frac{\theta(\bar{y}-x)}{\bar{y}}(\bar{x}-y)\right)\right. \\
& \left. + \frac{1}{2}([\gamma_\perp^\mu,\gamma_\perp^\nu])_{\alpha\beta}\left(\frac{\theta(x-\bar{y})\bar{x}}{yx}(\bar{x}-y-1)+\frac{\theta(\bar{y}-x)}{\bar{y}}(\bar{x}-y)\right)\right\} \\
& + \frac{\alpha_s \mathbf{C_F}}{4\pi}\,\bar{x}\,(\gamma_\perp^\mu\gamma_\perp^\nu)_{\alpha\beta}\,, \qquad (28)
\end{aligned}
$$

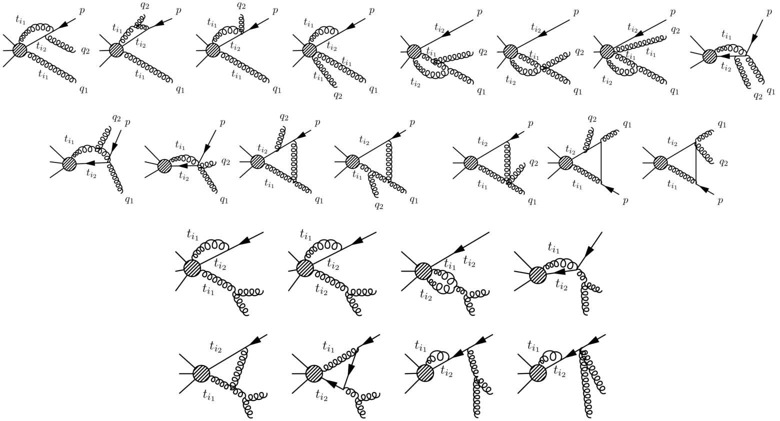
consistent with previous results [Hill, Becher, Lee, Neubert, 04', Beneke, Yang, 05'] and recent work [Alte, König, Neubert, 18'].

B2 to B2 mixing with F=1



$$
\begin{aligned}
&\gamma^{i}_{\mathcal{A}^\mu \partial^\nu \chi, \mathcal{A}^\rho \partial^\sigma \chi}(x,y) = \\
&g_\perp^{\mu\rho} g_\perp^{\nu\sigma} \frac{\alpha_s \mathbf{T}_{i_1} \cdot \mathbf{T}_{i_2}}{2\pi} \left\{ \theta(x-y) \left[\frac{1}{x-y}\right]_+ + \theta(y-x) \left[\frac{1}{y-x}\right]_+ \right. \\
&\left. - \theta(x-y) \frac{\bar{x}+\bar{y}}{\bar{y}^2} - \theta(y-x) \frac{x+2y}{2y^2} \right\} \\
&+ \frac{\alpha_s \mathbf{T}_{i_1} \cdot \mathbf{T}_{i_2}}{8\pi} M^{\mu\nu,\rho\sigma}(x,y) - \frac{\alpha_s(\mathbf{C_F} + \mathbf{T}_{i_1} \cdot \mathbf{T}_{i_2})}{8\pi} N^{\mu\nu,\rho\sigma}(x,y) \\
&+ \frac{\alpha_s \mathbf{C_F}}{8\pi} \frac{\bar{x}}{\bar{y}} (2 g_\perp^{\mu\nu} - x \gamma_\perp^\mu \gamma_\perp^\nu) \left(\gamma_\perp^\rho \gamma_\perp^\sigma + \frac{2\bar{y}}{y} g_\perp^{\rho\sigma} \right) \qquad (29)
\end{aligned}
$$

B2 to C2 mixing with F=1



$$\gamma^{i}_{\mathcal{A}^{\mu a}\partial^{\nu}\xi,\mathcal{A}^{\sigma d}\mathcal{A}^{\lambda e}\xi}(x,y_1,y_2) = -\frac{\alpha_s}{8\pi}\, I^{\mu\nu\sigma\lambda}_{ade}(x,y_1,y_2)\,. \qquad (30)$$

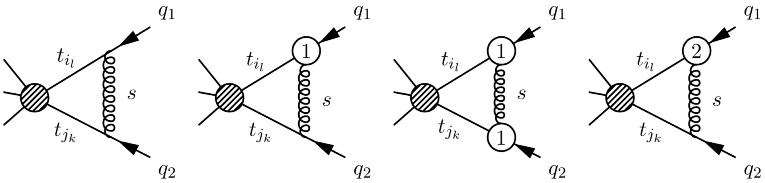
$$
\begin{aligned}
&I^{\mu\nu\sigma\lambda}_{ade}(x,y_1,y_2)|_{(c,i)_B} \\
&= \bar{x}\Bigg\{\Bigg[-\theta(x-y_2)\theta(\bar{y}_3-x)\frac{x^2\bar{y}_2+\bar{x}^2\bar{y}_3-\bar{y}_2\bar{y}_3}{\bar{y}_2 y_1\bar{y}_3} \\
&\quad +\theta(y_2-x)\theta(x)\frac{x^2}{y_2\bar{y}_3}+\theta(\bar{x})\theta(x-\bar{y}_3)\frac{\bar{x}^2}{\bar{y}_2 y_3}\Bigg]\Bigg[\frac{1}{2}g_\perp^{\nu\lambda}\left(\frac{\gamma_\perp^\sigma\gamma_\perp^\mu}{\bar{x}}+\frac{2g_\perp^{\mu\sigma}}{x\bar{x}}(\bar{x}-x)\right) \\
&\quad +\frac{1}{2}g_\perp^{\nu\sigma}\left(\frac{\gamma_\perp^\lambda\gamma_\perp^\mu}{\bar{x}}+\frac{g_\perp^{\mu\lambda}(2\bar{x}-x)}{x\bar{x}}\right)+\frac{1}{2}g_\perp^{\lambda\sigma}\left(\frac{\gamma_\perp^\nu\gamma_\perp^\mu}{\bar{x}}+\frac{2g_\perp^{\mu\nu}(\bar{x}+y_2)}{x\bar{x}}\right)+\frac{x-y_1}{2x\bar{x}}g_\perp^{\mu\lambda}\gamma_\perp^\nu\gamma_\perp^\sigma \\
&\quad -\frac{y_1}{2x\bar{x}}\left(g_\perp^{\nu\lambda}\gamma_\perp^\mu\gamma_\perp^\sigma+g_\perp^{\mu\nu}\gamma_\perp^\lambda\gamma_\perp^\sigma\right)-\frac{y_2}{2x\bar{x}}\left(g_\perp^{\mu\nu}\gamma_\perp^\sigma\gamma_\perp^\lambda+g_\perp^{\nu\sigma}\gamma_\perp^\mu\gamma_\perp^\lambda+g_\perp^{\mu\sigma}\gamma_\perp^\nu\gamma_\perp^\lambda\right)\Bigg] \\
&\quad +\frac{1}{2}\Bigg[\theta(x-y_2)\theta(\bar{y}_3-x)\frac{\bar{x}\bar{y}_3-x\bar{y}_2}{\bar{y}_2 y_1\bar{y}_3}+\theta(y_2-x)\theta(x)\frac{x}{y_2\bar{y}_3}-\theta(\bar{x})\theta(x-\bar{y}_3)\frac{\bar{x}}{\bar{y}_2 y_3}\Bigg] \\
&\quad \Bigg[-2g_\perp^{\nu\lambda}g_\perp^{\mu\sigma}+\frac{2y_2}{x}g_\perp^{\mu\nu}g_\perp^{\lambda\sigma}-g_\perp^{\mu\lambda}g_\perp^{\nu\sigma}-g_\perp^{\mu\lambda}\gamma_\perp^\nu\gamma_\perp^\sigma\frac{x-y_1}{\bar{x}}\Bigg]\Bigg\}f^{abe}f^{bcd}t^c+(y_1 d\sigma\leftrightarrow y_2 e\lambda)\,,
\end{aligned}
$$

Collinear anomalous dimensions with F=1

$$\gamma^i_{PQ} \quad = \quad \begin{array}{c||c|c} & J^{A1}_{\partial\chi} & J^{B1}_{\mathcal{A}\chi} \\ \hline\hline J^{A1}_{\partial\chi} & 0 & 0 \\ \hline J^{B1}_{\mathcal{A}\chi} & 0 & \gamma^i_{\mathcal{A}\chi,\mathcal{A}\chi} \end{array}$$

$$\gamma^i_{PQ} \quad = \quad \begin{array}{c||c|cc|cc} & J^{A2}_{\partial\partial\chi} & J^{B2}_{\mathcal{A}\partial\chi} & J^{B2}_{\partial(\mathcal{A}\chi)} & J^{C2}_{\mathcal{A}\mathcal{A}\chi} & J^{C2}_{\chi\bar{\chi}\chi} \\ \hline\hline J^{A2}_{\partial\partial\chi} & 0 & 0 & 0 & 0 & 0 \\ \hline J^{B2}_{\mathcal{A}\partial\chi} & 0 & (4.7) & (4.8) & (4.22) & (4.30) \\ J^{B2}_{\partial(\mathcal{A}\chi)} & 0 & 0 & (4.9) & 0 & 0 \\ \hline J^{C2}_{\mathcal{A}\mathcal{A}\chi} & 0 & 0 & 0 & (4.32) & (4.33) \\ J^{C2}_{\chi\bar{\chi}\chi} & 0 & 0 & 0 & (4.35) & (4.34) \end{array}$$

Soft anomalous dimensions

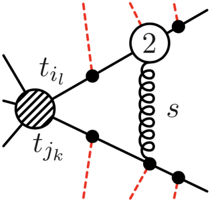


$$\gamma^{ij}_{\left(J^{T1}_{\chi,\xi}\right)_i \left(J^{T1}_{\chi,\xi}\right)_j, \left(J^{A1}_{\partial^\mu \chi}\right)_i \left(J^{A1}_{\partial^\nu \chi}\right)_j} = \frac{2\alpha_s}{\pi} \mathbf{T}_i \cdot \mathbf{T}_j \, G^{\mu\nu}_{ij} \,, \tag{31}$$

$$G^{\mu\nu}_{ij} \equiv \left(g^{\mu\nu} - \frac{n^\nu_{i-} n^\mu_{j-}}{n_{i-} n_{j-}} \right) \frac{1}{(n_{i-} n_{j-}) P_i P_j} \,. \tag{32}$$

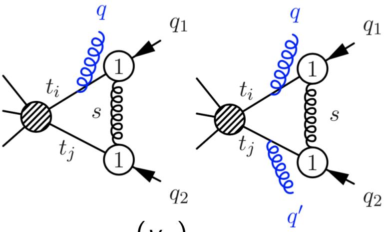
- The single insertions with $\mathcal{L}^{(1)}$ never contribute to the one-loop anomalous dimension matrix to $\mathcal{O}(\lambda^2)$.
- The soft one-loop diagrams within a single collinear direction do not contribute to the anomalous dimension at any power of λ.

One $\mathcal{L}^{(2)}$ insertion



$$\gamma^{ij}_{T(P,\mathcal{L}_k^{(2)}),Q} = 0 \qquad (33)$$

Two $\mathcal{L}^{(1)}$ insertions in two directions



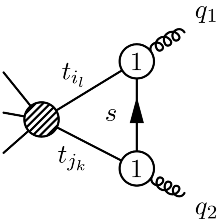
$$\gamma^{ij}_{\left(J^{T1}_{\chi\alpha,\xi}\right)_i \left(J^{T1}_{\chi\beta,\xi}\right)_j, \left(J^{B1}_{\mathcal{A}^{\mu}_b\chi\gamma}\right)_i \left(J^{A1}_{\partial^{\nu}\chi\delta}\right)_j}(y_{i_1})$$

$$= -\frac{\alpha_s}{\pi}\, \mathbf{T}^b_i\, (\mathbf{T}_i \cdot \mathbf{T}_j) G^{\lambda\nu}_{ij} \left(\gamma_{\lambda\perp i}\gamma^{\mu}_{\perp i} + \frac{\gamma^{\mu}_{\perp i}\gamma_{\lambda\perp i}}{\bar{y}}\right)_{\alpha\gamma} \delta_{\beta\delta} \tag{34}$$

$$\gamma^{ij}_{\left(J^{T1}_{\chi\alpha,\xi}\right)_i \left(J^{T1}_{\chi\beta,\xi}\right)_j, \left(J^{B1}_{\mathcal{A}^{\mu}_b\chi\gamma}\right)_i \left(J^{B1}_{\mathcal{A}^{\nu}_c\chi\delta}\right)_j}(y_{i_1}, y_{j_1})$$

$$= \frac{\alpha_s}{2\pi}\, \mathbf{T}^b_i \mathbf{T}^c_j (\mathbf{T}_i \cdot \mathbf{T}_j)\, G^{ij}_{\lambda\kappa} \left(\gamma^{\lambda}_{\perp i}\gamma^{\mu}_{\perp i} + \frac{\gamma^{\mu}_{\perp i}\gamma^{\lambda}_{\perp i}}{\bar{y}_{i_1}}\right)_{\alpha\gamma} \left(\gamma^{\kappa}_{\perp j}\gamma^{\nu}_{\perp j} + \frac{\gamma^{\nu}_{\perp j}\gamma^{\kappa}_{\perp j}}{\bar{y}_{j_1}}\right)_{\beta\delta} \tag{35}$$

Soft quark interaction



$$\gamma^{ij}_{T(P,\mathcal{L}^{(1)}_{k,\xi q},\mathcal{L}^{(1)}_{l,\xi q}),Q} = 0 \tag{36}$$

$$\gamma^{ij}_{T(P,\mathcal{L}^{(1)}_{k,\xi q}),Q} = 0 \tag{37}$$

The fermion number is conserved up to one-loop and $O(\lambda^2) \rightarrow$ classify the anomalous dimension according to collinear sectors with definite fermion number.

If the quark is massive and light compare to the hard scale, then there are large logarithms $\ln^n m^2/Q^2$.

A missing part

$$\mathbf{\Gamma} = -\gamma_{\rm cusp}(\alpha_s) \sum_{i<j} \mathbf{T}_i \cdot \mathbf{T}_j \ln\left(\frac{-s_{ij}}{\mu^2}\right) + \sum_i \gamma_i(\alpha_s) \tag{38}$$

Choose a non-special frame, $p_\perp \neq 0$,

$$\begin{aligned} s_{kj}^{(0)} &= \frac{1}{2}(n_{k-} n_{j-})(n_{k+} p_k)(n_{j+} p_j)\,, \\ s_{kj} &= s_{kj}^{(0)} + (n_{k+} p_k)(n_{k-} p_{\perp j}) + (n_{j+} p_j)(n_{j-} p_{\perp k}) + \mathcal{O}(\lambda^2)\,, \end{aligned} \tag{39}$$

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A missing part: Interpretation

$$J_i^{T1}(t_i) \to J_i^{A1\,\mu}(t_i)\,, \tag{40}$$

$$\gamma_{P_i Q_i}^{kj} = \frac{\alpha_s}{\pi}\,\mathbf{T}_k \cdot \mathbf{T}_j\,\frac{n_{j-}^{\mu}\,\delta_{ki}}{(n_{k-}n_{j-})(n_{i+}p_i)} + (k \leftrightarrow j)\,, \tag{41}$$

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RPI invariance implies

$$\int dsdt\, C^{(A0,A0)}(t,s)\,\bar{\chi}_j(sn_{j+})\Gamma\left[1+\frac{2t}{n_{i-}n_{j-}}\,n_{j-}\cdot\partial_{\perp i}\right]\chi_i(tn_{i+})\,,$$

$$C^{(A1,A0)\,\mu}(n_{i+}p_i,n_{j+}p_j) = \frac{-2n^\mu_{j-}}{n_{i-}n_{j-}}\frac{\partial}{\partial n_{i+}p_i}\,C^{(A0,A0)}(n_{i+}p_i,n_{j+}p_j)\,.$$

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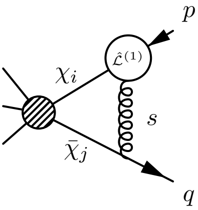
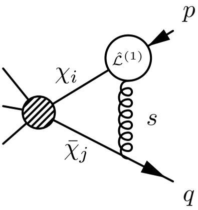
RG mixing

$$\begin{aligned}\frac{d}{d\ln\mu}\,C^{(A1,A0)\,\mu} \;&=\; -\left[\gamma_{\rm cusp}(\alpha_s)\mathbf{T}_i\cdot\mathbf{T}_j\ln\frac{-s^{(0)}_{ij}}{\mu^2}+\gamma_i\right]C^{(A1,A0)\,\mu}\\ &+\gamma_{\rm cusp}(\alpha_s)\mathbf{T}_i\cdot\mathbf{T}_j\frac{2n^\mu_{j-}}{(n_{i-}n_{j-})n_{i+}p_i}\,C^{(A0,A0)}(n_{i+}p_i,n_{j+}p_j)\end{aligned} \tag{42}$$

Revisit the soft loop

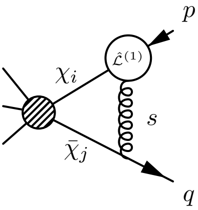




$$\mathcal{L}^{(1)}_{\xi_i} = \bar{\xi}_i \left(x^\mu_{\perp i} n^\nu_{i-} \, W_i \, g_s F^s_{\mu\nu} W^\dagger_i \right) \frac{\not{n}_{i+}}{2} \xi_i \,. \tag{43}$$

$$\hat{\mathcal{L}}^{(1)}_{\xi_i} = \bar{\xi}_i \left(i\not{D}_{\perp i} \frac{1}{i n_{i+} D_i} g_s \not{A}_{s\perp i} + g_s \not{A}_{s\perp i} \frac{1}{i n_{i+} D_i} i\not{D}_{\perp i} + [(x_{\perp i}\partial)(g_s n_{i-} A_s)] \right) \frac{\not{n}_{i+}}{2} \xi_i$$

Revisit the soft loop



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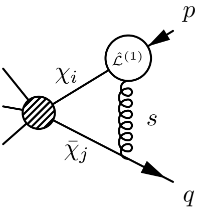
The difference is given by the eom Lagrangian

$$\Delta\mathcal{L}^{(1)}_{\xi_i} \equiv \hat{\mathcal{L}}^{(1)}_{\xi_i} - \mathcal{L}^{(1)}_{\xi_i} \quad = \quad \bar{\xi}_i \, ig_s x_{\perp i} A_s \frac{\delta S^{(0)}_{\xi_i}}{\delta \bar{\xi}_i} + \text{h.c.} \,, \tag{44}$$



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On-shell matrix element: take $p^2 \to 0$ first and then expand in ϵ, so $1/\epsilon (p^2)^{-\epsilon} \to 0$

To extract UV divergences: expand in ϵ first (and then $p^2 \to 0$)

Lessons learned

$$J_i^{T1}(t_i) \to J_i^{A1\,\mu}(t_i) \tag{45}$$

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EoM operators mixing into physical operators
$\rightarrow$ Violation of Kluberg-Stern-Zuber theorem (1975)?

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Which Lagrangian or field representation of SCET is the one that correctly reproduces QCD in the IR?

The KSZ theorem is violated due to the momentum derivative coming from the $x_\perp^\mu$ term in the Lagrangian, which in turn arises from the multipole expansion, and because there is a *logarithmic* dependence on p^2 instead of polynomial dependence since in SCET double poles in $1/\epsilon$ appear already at one loop.

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Once the counterterms and anomalous dimensions are determined, eom operators are no longer needed, field redefinition may be performed, and any Lagrangian obtained in this way can be used to compute *on-shell* amplitudes in SCET.

Double logarithms in off-diagonal splitting kernel

The DGLAP splitting kernel [Vogt 10']

$$P_{gq}^{\mathrm{LL}}(N) = \frac{1}{N}\frac{\alpha_s C_F}{\pi}\,\mathcal{B}_0(a), \qquad a = \frac{\alpha_s}{\pi}(C_F - C_A)\ln^2 N\,, \tag{46}$$

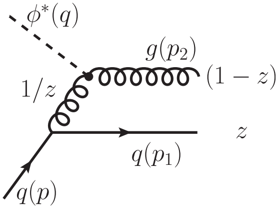
where

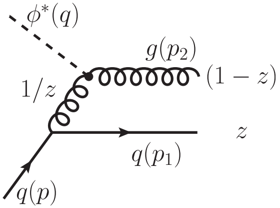
$$\mathcal{B}_0(x) = \sum_{n=0}^{\infty}\frac{B_n}{(n!)^2}x^n \quad , B_n = 1, \frac{-1}{2}, 0, \frac{1}{6}, 0, \frac{-1}{30}, 0, \frac{1}{42}\cdots \tag{47}$$

Compared to

$$P_{qq}^{\mathrm{LL}}(N) = -2\Gamma_{\mathrm{cusp}}(\alpha_s)\ln N \tag{48}$$

Off-diagonal DIS cross section





$$W_{\phi,q}\Big|_{q\phi^*\to qg} = \int_0^1 dz\,\left(\frac{\mu^2}{s_{qg}\,z\,\bar{z}}\right)^{\epsilon} \mathcal{P}_{qg}(s_{qg}\,,z)\Big|_{s_{qg}=Q^2\frac{1-x}{x}}\,, \qquad (49)$$

$$\mathcal{P}_{qg}(s_{qg}\,,z) \equiv \frac{e^{\gamma_E\epsilon}\,Q^2}{16\pi^2\Gamma(1-\epsilon)}\frac{|\mathcal{M}_{q\phi^*\to qg}|^2}{|\mathcal{M}_0|^2} = \frac{\alpha_s C_F}{2\pi}\frac{\bar{z}^2}{z} + \mathcal{O}(\epsilon,\lambda^2)$$

Off-diagonal DIS cross section

$$\begin{aligned}\mathcal{P}_{qg}(s_{qg}, z)|_{\text{1-loop}} &= \mathcal{P}_{qg}(s_{qg}, z)|_{\text{tree}} \frac{\alpha_s}{\pi} \frac{1}{\epsilon^2} \\ &\left(\mathbf{T}_1 \cdot \mathbf{T}_0 \left(\frac{\mu^2}{zQ^2} \right)^{\epsilon} + \mathbf{T}_2 \cdot \mathbf{T}_0 \left(\frac{\mu^2}{\bar{z}Q^2} \right)^{\epsilon} \right. \\ &\left. + \mathbf{T}_1 \cdot \mathbf{T}_2 \left[\left(\frac{\mu^2}{Q^2} \right)^{\epsilon} - \left(\frac{\mu^2}{zQ^2} \right)^{\epsilon} + \left(\frac{\mu^2}{zs_{qg}} \right)^{\epsilon} \right] \right) \quad (50)\end{aligned}$$

Off-diagonal DIS cross section

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We get the terms with $\mathbf{T}_1 \cdot \mathbf{T}_0$ and $\mathbf{T}_2 \cdot \mathbf{T}_0$ by standard method.

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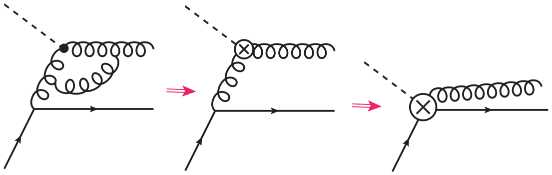
We get the terms with $\mathbf{T}_1 \cdot \mathbf{T}_0$ and $\mathbf{T}_2 \cdot \mathbf{T}_0$ by standard method.
Caution: Keep $z^{-\epsilon}$!

$$\frac{1}{\epsilon^2} \int_0^1 dz \, \frac{1}{z^{1+\epsilon}} \left(1 - z^{-\epsilon}\right) = -\frac{1}{2\epsilon^3} \tag{51}$$

$$\frac{1}{\epsilon^2} \int_0^1 dz \, \frac{1}{z^{1+\epsilon}} \left(\epsilon \ln z - \frac{\epsilon^2}{2!} \ln^2 z + \frac{\epsilon^2}{3!} \ln^3 z + \cdots \right) = -\frac{1}{\epsilon^3} + \frac{1}{\epsilon^3} - \frac{1}{\epsilon^3} + \cdots .$$

Off-diagonal DIS cross section

A new scale $\sqrt{z}Q$ emerges dynamically.



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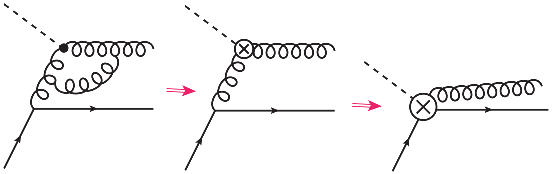

$$C^{A0}\left(Q^2, Q^2\right) \exp\left[-\frac{\alpha_s C_A}{2\pi}\frac{1}{\epsilon^2}\left(\frac{Q^2}{\mu^2}\right)^{-\epsilon}\right],$$

$$D^{B1}\left(zQ^2, zQ^2\right) \exp\left[-\frac{\alpha_s}{2\pi}\left(C_F - C_A\right)\frac{1}{\epsilon^2}\left(\frac{zQ^2}{\mu^2}\right)^{-\epsilon}\right]. \tag{52}$$

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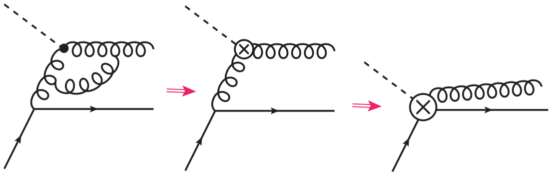

$$C^{A0}\left(Q^2,Q^2\right)\exp\left[-\frac{\alpha_s C_A}{2\pi}\frac{1}{\epsilon^2}\left(\frac{Q^2}{\mu^2}\right)^{-\epsilon}\right],$$

$$D^{B1}\left(zQ^2,zQ^2\right)\exp\left[-\frac{\alpha_s}{2\pi}\left(C_F-C_A\right)\frac{1}{\epsilon^2}\left(\frac{zQ^2}{\mu^2}\right)^{-\epsilon}\right]. \tag{52}$$

$$\mathcal{P}_{qg,\mathrm{hard}}=\frac{\alpha_s C_F}{2\pi}\frac{1}{z}\exp\left[\frac{\alpha_s}{\pi}\frac{1}{\epsilon^2}\left(-C_A\left(\frac{\mu^2}{Q^2}\right)^{\epsilon}+\left(C_A-C_F\right)\left(\frac{\mu^2}{zQ^2}\right)^{\epsilon}\right)\right],$$

Off-diagonal DIS cross section

$$W_{\phi,q}^{NLP,LP} = \frac{1}{2N\ln N}\frac{C_F}{C_F - C_A}\exp\left[\frac{\alpha_s C_F}{\pi}\frac{\ln N}{\epsilon}\right]\frac{w}{e^w - 1}\left(e^{a/w}e^{\widehat{S}_A} - e^{\widehat{S}_F}\right)$$

$$w \equiv -\epsilon \ln N, \qquad a = \frac{\alpha_s}{\pi}(C_F - C_A)\ln^2 N\,, \tag{54}$$

$$\widehat{S}_i = \frac{\alpha_s C_i}{\pi}\frac{1}{\epsilon^2}\left\{\left(\frac{\mu^2}{Q^2}\right)^{\epsilon}(N^\epsilon - 1) - \epsilon\ln N\right\}, \qquad i = A, F\,, \tag{55}$$

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$$F(w,a) \equiv \frac{we^{a/w}}{e^w - 1} = F_{\rm pole}(w,a) + F_{\rm fin}(w,a) \tag{56}$$

$$W_{\phi,q}^{NLP} = \tilde{C}_{\phi,q}^{NLP} Z_{qq}^{LP} + \tilde{C}_{\phi,g}^{LP} Z_{gq}^{NLP}\,, \tag{57}$$

Off-diagonal DIS cross section

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$$W^{NLP}_{\phi,q} = \tilde{C}^{NLP}_{\phi,q} Z^{LP}_{qq} + \tilde{C}^{LP}_{\phi,g} Z^{NLP}_{gq}\,, \tag{57}$$

$$P^{NLP,LL}_{gq}(N) = -\frac{1}{N}\frac{\alpha_s C_F}{\pi}\left[F_{\rm pole}(w,a) - w\frac{d}{da}F_{\rm pole}(w,a)\right] = \frac{1}{N}\frac{\alpha_s C_F}{\pi}\mathcal{B}_0(a)$$

Summary

- We list the subleading power operators for a N-jet process in SCET.
- Their anomalous dimensions have been calculated in the cases of fermion number $|F| = 1, 2$.
- The general structure has a similar pattern to the Leading Power result, but contains more information about the operators.
- At NLP, the KSZ theorem is violated.
- For singular operators, end-point divergences appear and naive factorization breaks down. New scales emerge and two-step matching is needed.
- Consistency relations in d-dimension are employed to derive the off-diagonal DGLAP splitting kernel, which contain double logarithms in itself.

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Thank you !